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Theory, analysis and design of high order reflective, absorptive filters

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Abstract: In this study, design methodology and equations necessary for low-loss high-order absorptive filters in the reflective-type configuration are introduced, derived and described in detail. Initially, a simple single, absorptive filter is described and its use in radio frequency front-ends is discussed. Based on this, improvements to the single, first-order absorptive filter are discussed and novel circuits that cater for lower loss higher-order absorptive filters are proposed. The necessary conditions for the achievement of the proposed higher-order absorptive filters from the proposed circuits are derived in order to establish the potential of the proposed circuits. As an experimental verification, first-, second- and third-order reflective-type absorptive filters are fabricated and their performances measured. In particular, in the case of a third-order absorptive filter, it is shown that the depth of the introduced attenuation is at least 27.4 dB across the frequency range of 2.17–2.27 GHz. Further, the use of the obtained absorptive filters was put to a test by inserting them in series with a commercially available band-pass filter.

1 Introduction

Filters are omnipresent in virtually all communication systems. As such, practical filter design still draws a great deal of attention from radio frequency (RF) engineers and academia, each focusing on various aspects of theoretical and practical filter designs.

The idea of absorptive filtering or leaky-wall filtering was first introduced in [1], and was implemented using waveguide components. In its simplest form, a leaky-wall filter comprises simply of a cascade of slots in a rectangular [2] or coaxial [3] waveguide, which couple to a single auxiliary waveguide with absorptive end-loads. Depending on the way absorption is introduced into the response of the main filter, absorptive filters can be clustered into two broad groups: transmission type absorptive filters and reflection type absorptive filters.

The main idea behind transmission type absorptive filters or dual (multiple) phase path cancellation filters lies in providing two or more different paths for the travelling wave signal, which, due to the differences in the electrical lengths of the paths, will be cancelled (absorbed by the parasitic losses of travelling wave paths) at the desired frequency at the output port. For example, in [4], a varactor diode is used to tune the position of the 70 MHz wide and 40 dB deep notch over a 9.5–10.5 GHz frequency span. The network comprised three paths, of which two are active, whereas in [5], an arbitrary selection of two passive networks is used to provide RF signal cancellation/absorption. Further, in [6] two notch filters positioned at the input and output of the main filter were used to inject a transmission zero into the stop-band of the main filter, at the expense of increased insertion loss. In [7] a parallel combination of a band-pass and a band-stop filter was used to increase the Q-factor of the notch by 325, while in [8], the same approach was implemented using a varactor diode in order to obtain a tunable transmission zero. The main drawback of these approaches lies with their restriction to narrow-band applications, with one notable exception in [9]. The rest of the literature on transmission-type absorptive filters focuses on the design of networks that are inherently wideband, reflection-less and consist of absorptive elements within the network. Examples of such work are found in [10, 11]. In particular, it was shown in [11] that the proposed networks can be used in the design of a variety of band-pass, band-stop, low-pass and high-pass filters.

Reflection-type absorptive filters are normally used when very low reflections are desired. For example, in [12, 13] it was shown that perfect matching can be achieved using lossy resonators, and that the values of the Q-factor of the overall reflective circuit were significantly greater than that achievable using the lossy resonators alone. In [14] a series tuned reflection load is used in the design of absorptive filters in order to increase the power handling capability. Furthermore, it was shown theoretically in [15–17] that reflective-type networks can be used in the design of band-pass filters.

In [18] a reflective-type network terminated in band-pass filters was proposed and designed in order to introduce increased attenuation in the stop-band of a transmit filter, by virtue of an additional, receive filter. Further advancements to the reflective-type, absorptive filter concept of [18] are reported in [19, 20], where a second-order, reflective-type, absorptive filter was introduced without the need of replicating 3-dB couplers (circulator). This was achieved by using pre-described values of either lumped elements [19] or quarter-wave transformers [20] suitably connected to band-pass filters in the circuit of the reflective loads.

This paper builds upon the concepts introduced in [19, 20] proposed earlier by the authors in order provide a generalisation of the concept of reflective-type absorptive filtering. In particular, this paper provides novel circuits that enable an arbitrary increase of the order of the reflection-type absorptive filter without replication of 3-dB couplers (circulators). The conditions imposed upon the proposed circuits in order to yield a higher order, reflective-type absorptive filtering response are derived in detail in order to present the potential of the proposed technique.

2 Theory and analysis

The absorptive filter in a reflective-type configuration of the first order is presented in Fig. 1*a*. The circuit comprises a 3-dB coupler, two band-pass filters, termed filters 2, connected to the through and coupled ports of the 3-dB coupler and two termination resistors, R. The operational principle is simple – band-pass filters 2 reject the majority of the RF power that falls in their stop-band, while dissipating the majority of RF power that falls within their pass-band across the termination resistors, R. The amount of RF power dissipated on the termination resistors is proportional to the reflection characteristics of filters 2. Under the assumption of an

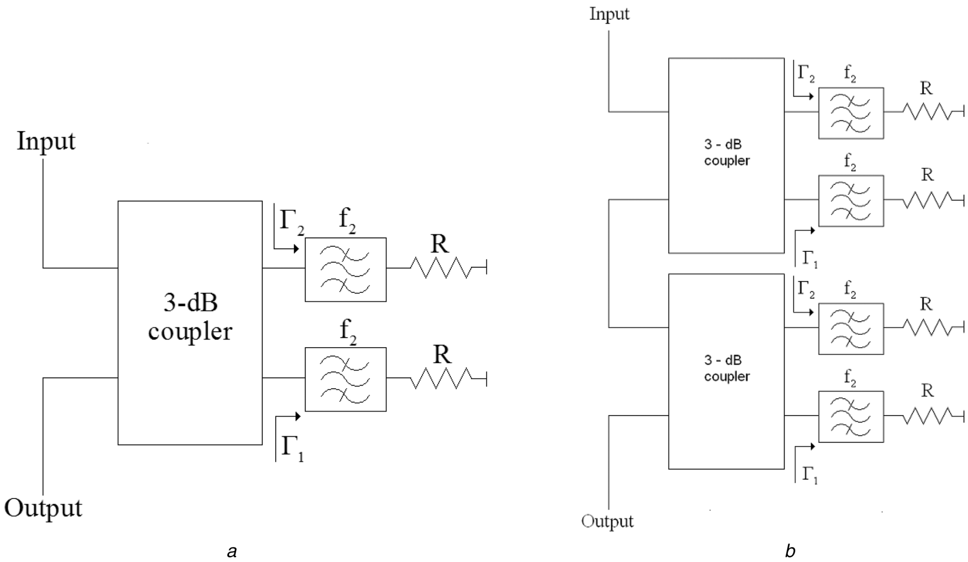


Fig. 1 *Simple broadband absorptive filters of*
(a) First order and ***(b)*** Second order

ideal 3-dB coupler, the expressions for the reflection and transmission coefficients of the structure of Fig. 1*a* become [21]:

$$S_{11} = 0.5(\Gamma_1 - \Gamma_2) \quad \text{and} \quad S_{21} = -\mathrm{j}0.5(\Gamma_1 + \Gamma_2) \tag{1}$$

If the reflective loads are the same, i.e. $\Gamma_1 = \Gamma_2 = \Gamma$, it follows that $S_{11} = 0$ and $S_{21} = -\mathrm{j}\Gamma$, i.e. the reflection coefficient of filters 2 now becomes the transmission coefficient of the overall absorptive filter.

The magnitude of the transmission coefficient of the absorptive filter of Fig. 1*a* can be expressed in terms of the input impedance of filters 2, *Z*, and the characteristic impedance of the 3-dB coupler (3-dB coupler loss included)

$$|S_{21}|_{\mathrm{dB}} = 20\log_{10}\left(\left|\frac{Z - Z_0}{Z + Z_0}\right|\right) + 2 \cdot L_{\mathrm{coup}},\ dB \tag{2}$$

where L_{coup} stands for the rated losses of the 3-dB coupler. It is important to realise that the insertion losses of an individual 3-dB coupler in the reflective-type configuration are two times greater than their rated values, since RF signals travel twice through the device – once to reach the reflective loads and second time to reach the respective input/output ports. The result of (2) has the following two implications; first, filters 2 can be lossy since it is their impedance matching characteristic that influences the amount of RF power absorption and not their insertion loss performance and second, due to the fact that the absorptive filter is usually used in series with a main filter that has an overlapping characteristics with filters 2, filters 2 can have a low power rating, inferring the use of a cheaper technology.

The amount of RF power absorption by the absorptive filter of Fig. 1*a* is limited by the achievable impedance matching of filters 2, i.e. its input impedance, *Z*. Usually, an impedance matching around 15–20 dB is considered satisfactory in most cases. A better impedance match is usually more difficult to obtain. With reference to Fig. 1*a*, if an attenuation depth greater than 15–20 dB is required, an alternative solution to improved impedance matching at filters 2 needs to be sought. One intuitive solution that increases the depth of attenuation, or in other words, increases the order of the reflective-type, absorptive filter, is in the form of replication of the circuit of Fig. 1*a*, shown in Fig. 1*b*. The mathematical expression for the magnitude of the transmission coefficient of this circuit is

$$\begin{aligned} |S_{21}|_{\mathrm{dB}} = 20\log_{10}&\left(\left|\frac{(Z - Z_0)(Z - Z_0)}{(Z + Z_0)(Z + Z_0)}\right|\right) \\ &+4 \cdot L_{\mathrm{coup}}, \mathrm{dB} + L_{\mathrm{line}},\ \mathrm{dB} \end{aligned} \tag{3}$$

while its reflection coefficient is maintained at $|S_{11}|_{\mathrm{lin}} = 0$.

Here, L_{line} stands for the insertion losses of the line connecting the two first-order absorptive filters of Fig. 1*b*. The first term on the right-hand side of (3) states that a double zero and hence the doubling of attenuation in the pass-band of filters 2 as compared to the first-order, reflective-type absorptive circuit of Fig. 1*a* is achieved. The last two terms in (3) represent additionally introduced insertion losses. They are particularly detrimental outside the pass-band of filters 2, as they increase the insertion losses in the pass-band of the overall absorptive filter.

In general, a cascade connection of an arbitrary number, *n*, of the first-order reflective-type, absorptive filters of Fig. 1*a* will result in the following expression for the magnitude of the overall transmission coefficient:

$$\begin{aligned} |S_{21}|_{\mathrm{dB}} = 20\log_{10}&\left(\left|\frac{(Z - Z_0)^n}{(Z + Z_0)^n}\right|\right) + 2 \cdot n \cdot L_{\mathrm{coup}}, \mathrm{dB} \\ &+(n-1) \cdot L_{\mathrm{line}}, \mathrm{dB} \end{aligned} \tag{4}$$

The last two terms in (4) show that a simple replication of first-order reflective-type, absorptive filters for the achievement of a higher-order reflective-type, absorptive filter, results in an increased insertion loss in its pass-band. This increase of insertion loss scales directly with the increase of the filter order and the number of 3-dB couplers. As such, the importance of reducing the number of 3-dB couplers for the achievement of high-order reflective-type, absorptive filters becomes paramount. The next section presents ways to increase the depth of absorptive attenuation and hence the order of the reflective-type, absorptive filters without the introduction additional 3-dB couplers.

2.1 Second-order absorptive circuit

A second-order absorptive filter can be achieved with the reflective circuit of Fig. 2*a*, [21]. The reflective circuit consists of filters 2 and impedance transformers with characteristic impedances k_{11} and k_{12}. The conditions needed for the achievement of the second-order absorptive filter are derived from the expression for the magnitude of the transmission coefficient of the reflective circuit of Fig. 2*a*, given by

$$|S_{21}|_{\mathrm{dB}} = 20\log_{10}\left(\left|\frac{-Zk_{11}^2 + Z_0k_{12}^2 + Z_0Z^2}{Zk_{11}^2 + Z_0k_{12}^2 + Z_0Z^2}\right|\right) + 2 \cdot L_{\mathrm{coup}},\ \mathrm{dB} \tag{5}$$

where k_{11} and k_{12} represent the characteristic impedance of the impedance transformers depicted in Fig. 2*a*.

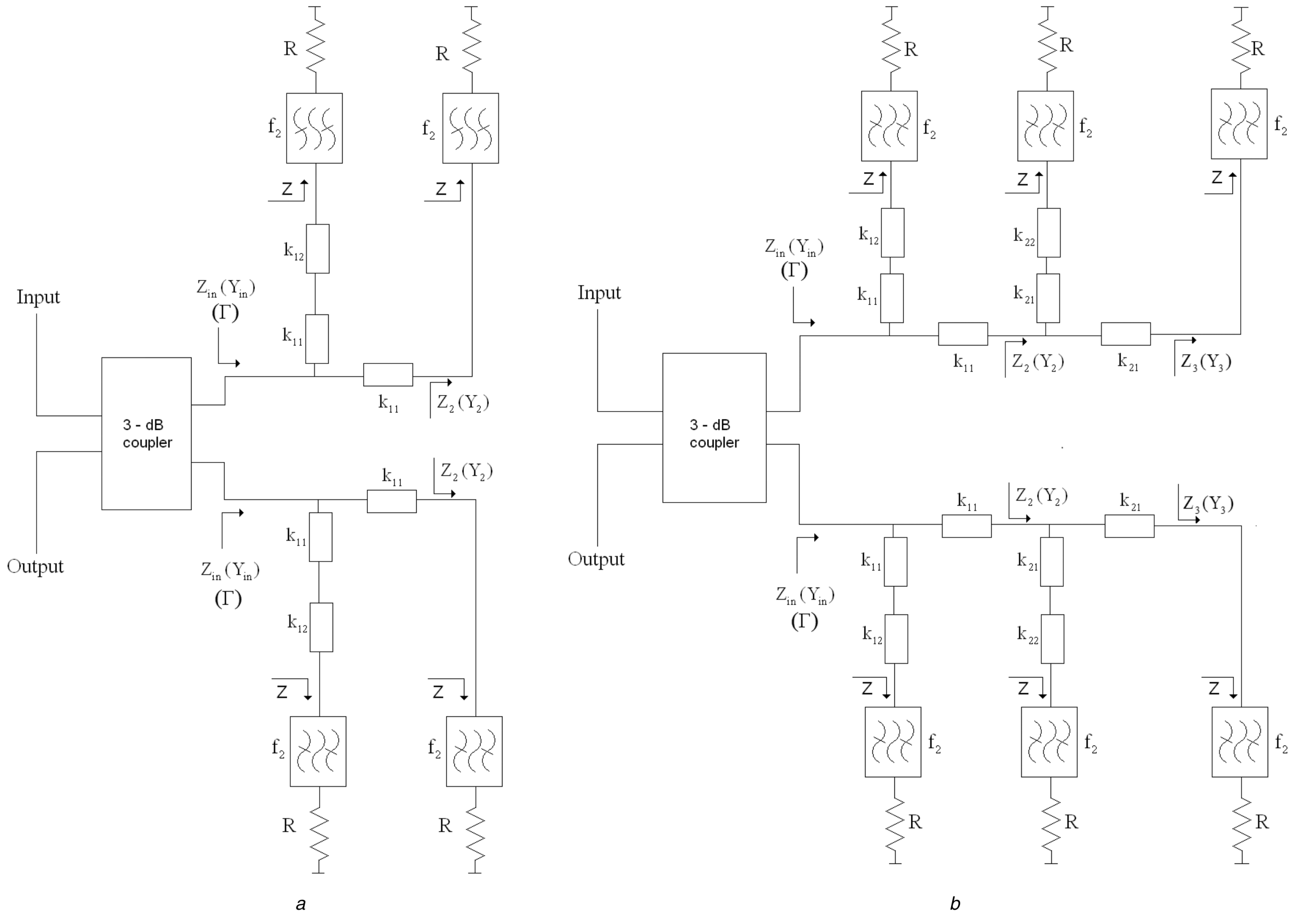


Fig. 2 *Circuit for proposed*
(a) Second-order reflective-type absorptive filter and, ***(b)*** Third-order reflective-type absorptive filter

Ignoring the losses in the 3-dB coupler and setting $|S_{21}|_{\text{lin}} = 0$, the conditions for the introduction of a second-order absorptive filter response are obtained. This yields a quadratic equation in Z, which has the following solution:

$$Z_{1,2} = \frac{k_{11}^2 \pm \sqrt{k_{11}^4 - 4Z_0^2 k_{12}^2}}{2Z_0} \tag{6}$$

The condition that the discriminant in (6) is zero yields a double and real zero at

$$Z_{1,2} = \frac{k_{11}^2}{2Z_0} \tag{7}$$

while the condition for zero discriminant yields the following relationship between the characteristics impedances of the impedance transformers:

$$Z_0 = \frac{k_{11}^2}{k_{12}} \tag{8}$$

The magnitude of the transmission coefficient of (5) now becomes

$$|S_{21}|_{\text{dB}} = 20\log_{10}\left(\left|\frac{(Z - Z_0)(Z - Z_0)}{(Z + Z_0)(Z + Z_0)}\right|\right) + 2 \cdot L_{\text{coup}}, \text{dB} \tag{9}$$

Equation (9) shows that a double zero of (6) and hence the second-order, reflective-type, absorptive filter response can be achieved using the proposed circuit of Fig. 2*a*, without replicating the number of 3-dB couplers. A comparison of (9) with (3) reveals that (9) offers lower insertion losses in the pass-band of the overall absorptive filter, by the theoretical amount of $2 \cdot L_{\text{coup}} + L_{\text{line}}$, without sacrificing the benefits of increased attenuation depth in the pass-band of filters 2 and reducing the filter's component count.

2.2 Third-order absorptive circuit

A third-order absorptive filter is achievable with the circuit of Fig. 2*b*. In this case, the reflective circuit consists of filters 2 and impedance transformers with characteristic impedances, k_{11}, k_{12}, k_{21} and k_{22}. The magnitude of the transmission coefficient of the reflective circuit of Fig. 2*b*, is given by

$$|S_{21}|_{\text{dB}} = 20\log_{10}\left(\left|\frac{aZ^3 + bZ^2 + cZ + d}{-aZ^3 + bZ^2 - cZ + d}\right|\right) + 2 \cdot L_{\text{coup}},\ \text{dB} \tag{10}$$

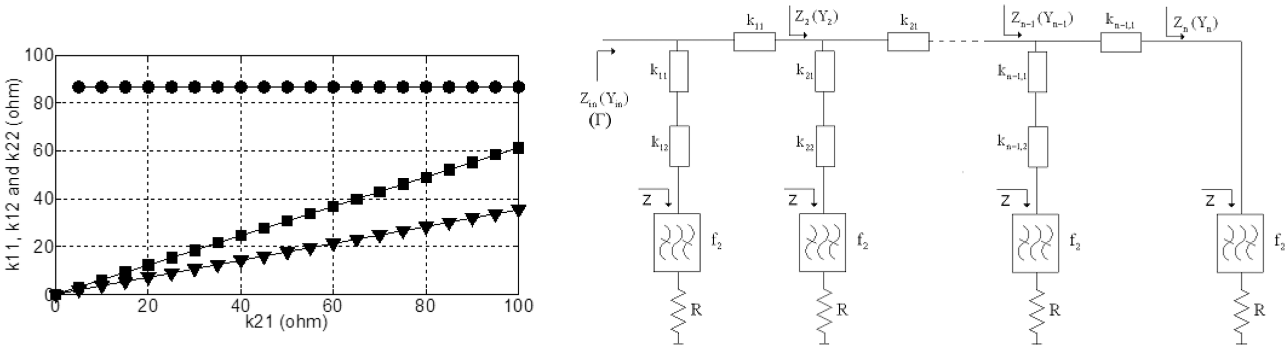
where $a = -k_{11}^2$, $b = Z_0 k_{12}^2 + Z_0 k_{21}^2$, $c = -k_{11}^2 k_{22}^2$ and $d = Z_0 k_{12}^2 k_{22}^2$. Ignoring the losses in the 3-dB coupler and setting $|S_{21}|_{\text{lin}} = 0$, the zero transmission condition is achieved. In this case, a third-order polynomial in Z is obtained and needs to be solved so that it has a single multiple and real root. This is accomplished by setting the discriminant, Δ, of the nominator of the third-order polynomial of (10) to be zero.

$$\Delta = 18abcd - 4b^3 d + b^2 c^2 - 4ac^3 - 27a^2 d^2 = 0 \tag{11}$$

The condition that the discriminant (11) is zero yields a triple zero at

$$Z = -\frac{b}{3a} = \frac{Z_0 k_{12}^2 + Z_0 k_{21}^2}{3k_{11}^2} \tag{12}$$

Solving systematically (11) and (12), yields the following conditions for the characteristic impedances, k_{12}, k_{11} and k_{22}



a b

Fig. 3 *Dependence of the characteristic impedances and generalisation of the reflective loads*
(a) Variation of quarter-wave characteristic impedances k_{12} (triangles), k_{11} (squares) and k_{22} (circles) against k_{21} for $Z_0 = 50\,\Omega$ and, ***(b)*** Generalisation of reflective load required for proposed *n*th order, reflective-type absorptive filters

$$k_{12}^2 = \frac{1}{8}k_{21}^2; \quad k_{11}^2 = \frac{3}{8}k_{21}^2 \quad \text{and} \quad k_{22} = \frac{\sqrt{27}}{3}Z_0 \tag{13}$$

where Z_0 and k_{21} are used as parameters. Since, Z_0 is usually, but not necessarily, $50\,\Omega$, only k_{21} can be used in the adjustment of the rest of the impedances of the impedance transformers, k_{12}, k_{11} and k_{22}. A full derivation of (13) from (11) and (12) is given in the Appendix. The transmission coefficient of (10) now becomes

$$\left|S_{21}\right|_{\text{dB}} = 20\log_{10}\left(\left|\frac{(Z-Z_0)(Z-Z_0)(Z-Z_0)}{(Z+Z_0)(Z+Z_0)(Z+Z_0)}\right|\right) + 2\cdot L_{\text{coup}},\ \text{dB} \tag{14}$$

Equation (14) indicates that the third-order, reflective-type, absorptive filter response can be achieved without replicating the number of reflective circuits given in Fig. 1*a*. It also shows that the proposed circuit of Fig. 2*b* offers lower pass-band insertion losses compared to the pass-band insertion losses of an absorptive filter obtained by a simple replication of three first-order reflection-type circuits by the theoretical amount of $4\cdot L_{\text{coup}} + 2\cdot L_{\text{line}}$.

Fig. 3*a* depicts the dependence of the characteristic impedances k_{12}, k_{11} and k_{22} on k_{21} for the case when $Z_0 = 50\,\Omega$. Even though k_{22} does not vary with k_{21}, it was displayed in Fig. 3*a* for comparison with k_{12} and k_{11}.

2.3 Higher-order absorptive circuit

The extension of the proposed absorptive filtering concept to the fourth and higher orders is presented here. To do so, let us consider Fig. 3*b*, which shows a generalised view of a reflective load of the proposed absorptive filter.

The input admittance of the proposed circuit can be represented as

$$Y_{\text{in}} = \frac{k_{12}^2}{k_{11}^2}Y + \frac{1}{k_{11}^2 Y_2} \tag{15}$$

where

$$Y_2 = \frac{k_{22}^2}{k_{21}^2}Y + \frac{1}{k_{21}^2 Y_3}, \quad Y_3 = \frac{k_{32}^2}{k_{31}^2}Y + \frac{1}{k_{31}^2 Y_4}\cdots \tag{16}$$

or, in general

$$Y_{n-1} = \frac{k_{n-1,2}^2}{k_{n-1,1}^2}Y + \frac{1}{k_{n-1,1}^2 Y_n}, \quad \text{where} \quad Y_n = Y \tag{17}$$

Here, $k_{i,j}, i = 2,\ldots,n,\ j = 1,2$ represent the impedance transformers, n represents the order of the absorptive filter and $Y = Z^{-1}$. It can be inferred from (15)–(17) that the input admittance, Y_{in}, can be represented in the form of a generalised continued fraction

$$Y_{\text{in}} = \frac{k_{12}^2}{k_{11}^2}Y + \frac{1}{((k_{11}^2 k_{22}^2)/k_{21}^2)Y + (k_{11}^2/(((k_{21}^2 k_{32}^2)/k_{31}^2)Y + \cdots))} \tag{18}$$

or, equivalently

$$Y_{\text{in}} = b_0 + \frac{a_1}{b_1 + (a_2/(b_2+\cdots))} = b_0 + \underset{0}{\overset{m}{K}}\frac{a_k}{b_k}, \quad k = 1,\ldots,m,\ m = n-1 \tag{19}$$

where $b_0 = (k_{12}^2/k_{11}^2)Y$, $a_1 = 1$, $b_k = ((k_{k,1}^2 k_{k+1,2}^2)/k_{k,1}^2)Y,\ \forall n \geq 2,\ k = 1,\ldots,n-1$ and $a_k = k_{k-1,1}^2, \forall n \geq 3,\ k = 2,\ldots,n-1$.

Solving (19), and ignoring losses in the 3-dB coupler at this stage, one obtains the *n*th order admittance polynomial from which the expression for the *n*th order polynomial expression for the transmission coefficient can be derived

$$S_{21} = \frac{Y_0 - Y_{\text{in}}}{Y_0 + Y_{\text{in}}} \tag{20}$$

where Y_0 is the characteristic admittance of the 3-dB coupler.

Substituting (19) into (20) and using the transformation $Y_0 = Z_0^{-1}$ one obtains the expression for the transmission coefficient, S_{21}, as a function of impedance parameters.

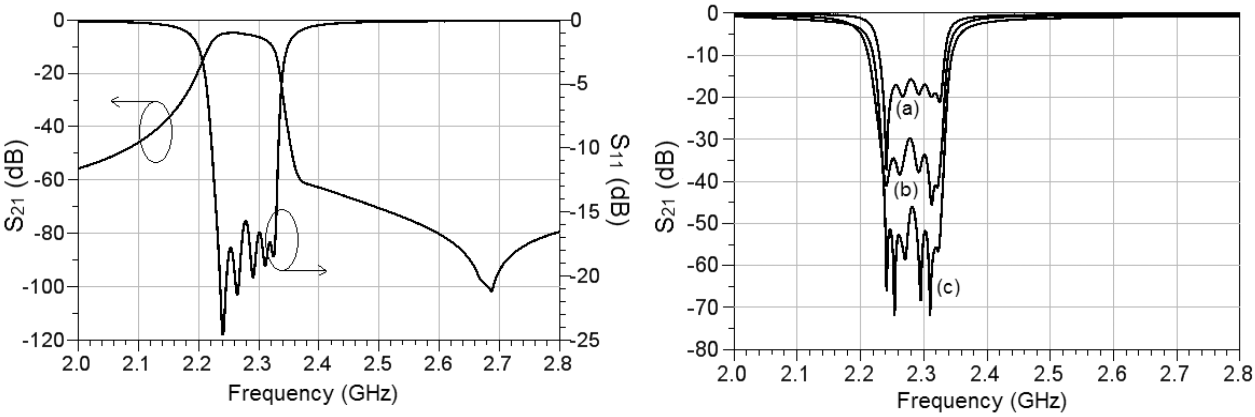
The *n*th order identical and real zeroes of such a polynomial yield the expressions for the transformer impedances, $k_{i,j},\ i = 2,\ldots,n,\ j = 1,2$. Taking 3-dB coupler losses into account, (20) becomes

$$\left|S_{21}\right|_{\text{dB}} = 20\log_{10}\left|\frac{(Z-Z_0)^n}{(Z+Z_0)^n}\right| + 2\cdot L_{\text{coup}},\ \text{dB} \tag{21}$$

Comparing (21) and (4) one can infer that the reduction of insertion losses in the pass-band of the obtained general and arbitrary order of the reflective-type absorptive filter as compared to a simple replication of an arbitrary number of first-order reflective-type absorptive filters is equal to

$$\Delta S_{21} = 2\cdot(n-1)\cdot L_{\text{coup}},\ \text{dB} + (n-1)\cdot L_{\text{line}},\ \text{dB} \tag{22}$$

In reality, this amount will be somewhat lower and dependent on the bandwidth of filters 2, due to the frequency dependent transformation of the impedance transformers across the pass-band of filters 2. Further, one must bear in mind that polynomials of order 5 and above cannot be uniquely, analytically solved for real



a b

Fig. 4 *In-house microstrip seventh-order combline Chebyshev filter fabricated on a Roger Duroid TMM3*
(a) Simulated *S*-parameters of seventh-order in-house made Chebyshev filter and ***(b)*** Simulated transmission coefficients of absorptive filters of (trace a) first, (trace b) second and (trace c) third orders

and identical zeroes. As such, for such high absorptive filter orders one must seek a numerical approach to this problem.

To complete the presented theory for the design of the whole filter chain, it is instructive to consider a cascade connection of the proposed absorptive filter and an arbitrary, band-pass filter. In this way, the ability of the proposed absorptive filter to introduce broad-band zeros of high order will be put to a test, Fig. 5*a*. In this regard, let us assume that the arbitrary, band-pass filter is a passive and reciprocal device with $S_{11\,\mathrm{B}} = S_{22\,\mathrm{B}}$ and $S_{12\,\mathrm{B}} = S_{21\,\mathrm{B}}$ and that the absorptive filter is also reciprocal and fully represented by its transmission coefficient, $S_{21\,\mathrm{A}}$, since in the case of matched reflective loads $S_{11\,\mathrm{A}} = S_{22\,\mathrm{A}} = 0$. The transmission matrix of this cascade connection now becomes

$$T = \begin{bmatrix} \dfrac{1}{S_{21\,\mathrm{B}}S_{21\,\mathrm{A}}} & -\dfrac{S_{11\,\mathrm{B}}S_{21\,\mathrm{A}}}{S_{21\,\mathrm{B}}} \\ -\dfrac{S_{11\,\mathrm{B}}}{S_{21\,\mathrm{B}}S_{21\,\mathrm{A}}} & \dfrac{S_{21\,\mathrm{B}}^2 - S_{11\,\mathrm{B}}^2}{S_{21\,\mathrm{B}}}S_{21\,\mathrm{A}} \end{bmatrix} \tag{23}$$

which corresponds to the following *S*-parameters:

$$S = \begin{bmatrix} S_{11\,\mathrm{B}} & S_{21\,\mathrm{B}}S_{21\,\mathrm{A}} \\ S_{21\,\mathrm{B}}S_{21\,\mathrm{A}} & S_{11\,\mathrm{C}}S_{21\,\mathrm{B}}^2 \end{bmatrix} \tag{24}$$

or in the log scale the *S*-parameters become

$$\begin{aligned} S_{11} &= 20\log_{10}\left|S_{11\,\mathrm{B}}\right| \\ S_{12} = S_{21} &= 20\log_{10}\left|S_{21\,\mathrm{A}}\right| + 20\log_{10}\left|S_{21\,\mathrm{B}}\right| \\ S_{22} &= 20\log_{10}\left|S_{21\,\mathrm{B}}\right| + 40\log_{10}\left|S_{21\,\mathrm{A}}\right| \end{aligned} \tag{25}$$

In (23)–(25), all indices with A refer to the absorptive filter in general, while indices with B correspond to an arbitrary, band-pass filter. Equation (25) demonstrates that the design of a complete, cascaded filter consisting of a series connection of an arbitrary band-pass filter and an absorptive filter can be broken down into two separate filter designs – one arbitrary band-pass and one general absorptive filter. This information could be used by RF filter designers to produce an exact tailor-made frequency response. As an example, finite frequency transmission zeroes of an arbitrary order can be introduced by an absorptive filter, whereas the pass-band of the complete filter characteristics is influenced by the design of the band-pass filter. The design of a complete filter with an equalised group delay is not entirely straightforward though, since it would require the minimisation of the first derivative of S_{21} which is composed of two terms – one coming from the absorptive filter and the other coming from the arbitrary band-pass filter. Careful co-design procedures need to be developed in that case.

3 Results

3.1 Simulated results

To test the theory presented in the previous section, several absorptive filters are designed and simulated. For this purpose, an in-house microstrip seventh-order comb line Chebyshev filter fabricated on a Roger Duroid TMM3 [22] substrate is used as filter 2, and its simulated *S*-parameters are shown in Fig. 4*a*. This filter exhibits a −15 dB bandwidth of about 100 MHz, located between 2.23 and 2.33 GHz with an insertion loss of about 5.4 dB in the middle of its pass-band of 2.28 GHz.

By assuming $Z_0 = 50\,\Omega$, the values of the quarter-wave impedance transformers necessary for the absorptive filter of the second order are obtained from (7) and (8) to yield

$$k_{11} = 70.7\,\Omega \quad \text{and} \quad k_{12} = 50\,\Omega \tag{26}$$

Next, in order to obtain the values of the impedance transformers necessary for the absorptive filter of third order, one needs to set parameter k_{21} in such a way so that its dependent parameters, k_{11}, k_{12} and k_{21} attain physically realisable values. From Fig. 3*a*, one can see that a value of $k_{21} = 80\,\Omega$, yields realisable values for the rest of the impedance transformers. In particular, the use of (13) yields $k_{12} = 28.28\,\Omega$, $k_{11} = 49\,\Omega$ and $k_{22} = 86.6\,\Omega$, all of which were designed on Roger Duroid TMM3 substrate with a height of 0.635 mm [22]. As a 3-dB coupler, a quadrature surface mount hybrid coupler from Anaren [23], 1P603S, is used. According to manufacturer's data, this device operates across the frequency range of 2.3–2.7 GHz, with a maximum insertion loss of 0.3 dB. However, the measured *S*-parameters of the device indicate that if a greater amplitude imbalance can be tolerated, the device is operable from 2.1 to 2.9 GHz.

In this case, the amplitude imbalance is increased from 0.3 to 0.37 dB while insertion loss remains 0.3 dB. The simulated transmission coefficients of the absorptive filters of first-, second- and third-orders using the in-house seventh-order Chebyshev filter as filter 2 are shown in Fig. 4*b*. The reflection coefficients are not presented since, across the frequency range indicated in Fig. 4*b* they are better than −20 dB.

The results of Fig. 4*b* indicate that the depth of absorption scales with the increase of the order of the absorptive filter. In particular, the first-order absorptive filter has introduced attenuation of over 15 dB in the band of frequencies corresponding to the pass-band of the in-house designed Chebyshev filter, as shown in Fig. 4*a*. Higher-order absorptive filters, namely, second- and third-order have increased the amount of attenuation in line with theoretical predictions. In particular, the second-order

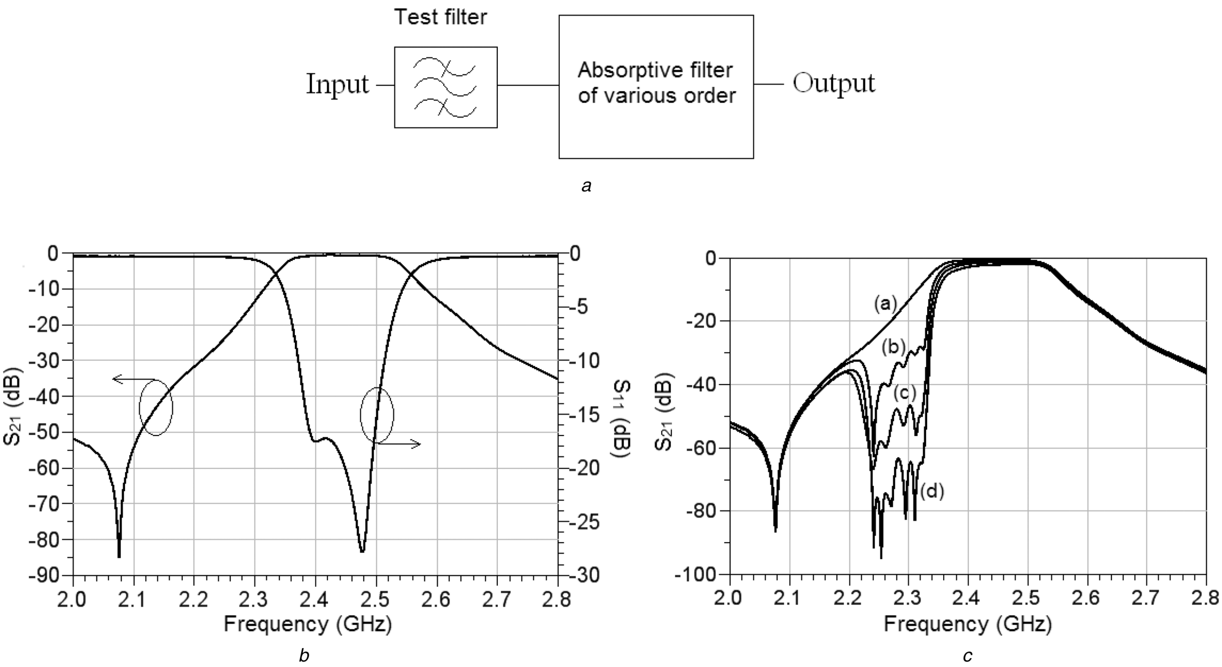


Fig. 5 *Designed absorptive filters are tested for their ability to introduce increased absorption into the stop-band response of a test filter*
(a) Cascade connection of test filter and absorptive filters of various order, ***(b)*** Simulated *S*-parameters of test filter – CER0367A and, ***(c)*** Simulated transmission coefficients of CTS filter (CER0367A) (trace a), cascade connection of test filter with (trace b) first-order absorptive filter, (trace c) second-order absorptive filter and (trace d) third-order absorptive filter

Table 1 Simulated performance of complete filter of Fig. 5*a* (test filter connected in cascade with various absorptive filters)

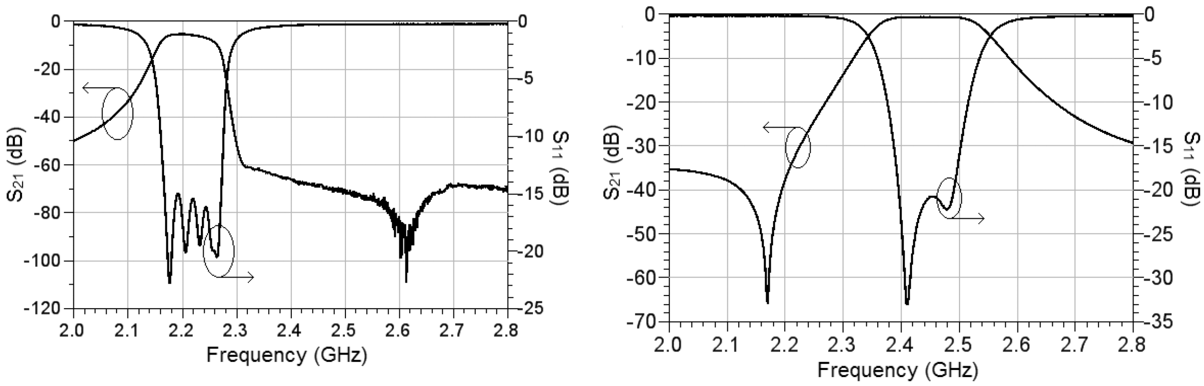
	Minimum attenuation, dB	Maximum increase of insertion loss, dB
test filter in cascade with first-order absorptive filter	15.6	0.7
test filter in cascade with proposed second-order absorptive filter	29.65	1.16
test filter in cascade with second-order absorptive filter obtained by cascade connection of two first-order absorptive filters	31.23	1.6
test filter in cascade with proposed third-order absorptive filter	46	2.16
test filter in cascade with third-order absorptive filter obtained by cascade connection of three first-order absorptive filters	46.9	2.51

absorptive filter has introduced an additional attenuation of about 15 dB compared to the first-order absorptive filter in the same frequency range, while the third-order absorptive offers a further 15 dB increase in attenuation. Next, the designed absorptive filters are tested for their ability to introduce increased absorption into the stop-band response of a test filter, as schematically shown in Fig. 5*a*.

For this purpose, the proposed absorptive filters of the first-, second- and third-orders are connected in cascade with the test filter and the overall response of the structure formed in this way is observed. The test filter of Fig. 5*a* is a 3-pole filter from CTS, CER0367A [24], which according to manufacturer's data, has a pass-band between 2.4 and 2.5 GHz. Its *S*-parameters obtained from the manufacturer's data are shown in Fig. 5*b*. The choice for this particular filter was dictated by the need for a filter that exhibits overlapping frequency characteristics with the main filter. The simulated transmission coefficient of the filter structure of Fig. 5*a* for the cases when the absorptive filters are of first, second and third order is shown Fig. 5*c*. The cascade connection of the test CER0367A filters with absorptive filters of the first, second and third order has resulted in a significant increase of attenuation in the stop-band of the CER0367A filter. This increase is commensurate with the increase offered by the absorptive filters alone, as depicted in Fig. 4*b*. However, the increase of attenuation in the stop-band of the test filter has not been achieved without a penalty. Table 1 summarises the results. It is obvious from this table that the proposed absorptive filters have increased the amount of attenuation in the stop-band of the CER0367A filter, however, higher-order absorptive filters have also increased the insertion loss as compared to the losses exhibited by the test, CER0367A filter. In particular, the first-order absorptive filter has increased the insertion loss by 0.7 dB, second-order absorptive filter by 1.16 dB and third order by 2.16 dB. Table 1 also shows the amount of attenuation that is achieved for the case when higher-order absorptive filters are not realised using the presented theory, but by a simple cascade connection of absorptive filters of the first order. In this case, the interconnecting microstrip lines needed for the second- and third-order absorptive filters are assumed to have an insertion loss of 0.2 dB. From here one can appreciate that while the proposed absorptive filters offer a similar attenuation in the stop-band as their cascade connection counterparts, they offer lower insertion losses. The simulated results presented in this section have been obtained by using Advanced Design System (ADS) from Agilent [25].

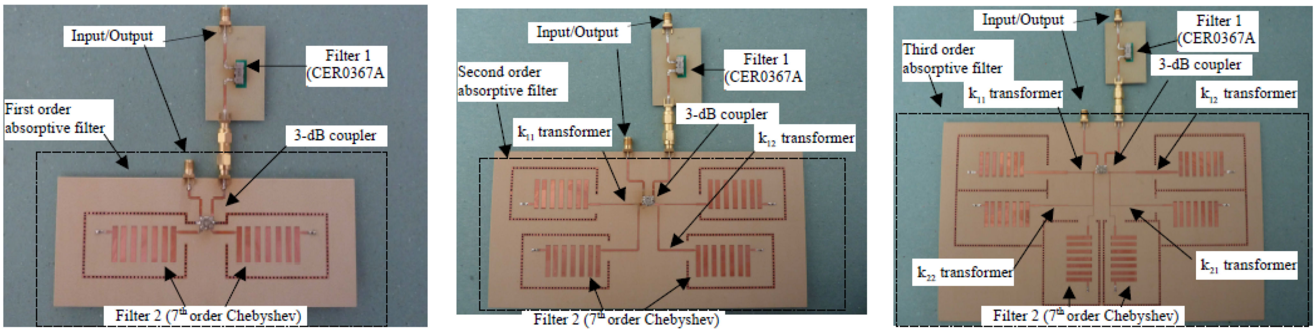
3.2 Measured results

Based on the theory and the simulated results in the previous sections, first-, second- and third-order absorptive filters were fabricated. An in-house designed seventh-order Chebyshev filter used as filter 2 has been fabricated and its *S*-parameters are shown in Fig. 6*a*, whereas the measured *S*-parameters of the test filter, CER0367A are shown in Fig. 6*b*. The measured *S*-parameters of the test filter, CER0367A, show a good agreement with the .s2p parameters supplied by the manufacturer, while the measured *S*-parameters of the in-house designed seventh-order Chebyshev filter show a downward frequency shift. In particular, the measured −15 dB reflection coefficient bandwidth of this filter lies between 2.17 and 2.27 GHz, whereas the simulated −15 dB reflection coefficient bandwidth is between 2.23 and 2.33 GHz. Further, insertion loss of



a b

Fig. 6 *Measured S-parameters of*
(a) Seventh-order in-house made Chebyshev filter and ***(b)*** Test filter – CER0367A



a b c

Fig. 7 *Photographs of*
(a) First-order, ***(b)*** Second-order and, ***(c)*** Third-order absorptive filters connected in cascade with test CER0367A filter

the fabricated in-house filter is also increased and now stands at ~6 dB at the frequency corresponding to the middle of its pass-band, 2.22 GHz.

The measured performance of the proposed absorptive filters, shown in Fig. 7 is recorded for two configurations: first, the proposed absorptive filters are measured in the stand-alone configuration and second, the performances of the cascade connections of the proposed absorptive filters with the test, CER0367 A filter are measured.

Fig. 8*a*, (trace a), represents the transmission coefficient of the first-order absorptive filter, which, as predicted by (1) represents the reflection coefficient of in-house made filter 2, whose response is depicted in Fig. 6*a*. There are, however, some differences between the two responses. For example, it can be inferred from Fig. 6*a* that the minimum reflection coefficient of the in-house made filter 2 is about 15 dB, while the minimum transmission coefficient of the circuit of Fig. 8*a* (trace a), is about 11.87 dB in the frequency range 2.17–2.27 GHz, i.e. the pass-band of filter 2. This could be attributed to the non-perfect manufacturing of the printed circuit boards. Fig. 8*a* (trace b), represents the transmission coefficient of the proposed second order absorptive filter, which, according to (7) and (8) leads to doubling of the transmission coefficient of Fig. 8*a* (trace a). The response of Fig. 8*a* (trace b) largely confirms this fact, where the minimum depth of the transmission coefficient in the frequency range of the operation of the in-house made filter 2, is about 23.5 dB, which corresponds well to the doubling of the attenuation offered by the first-order absorptive filter, 23.7 dB, as theoretically predicted. It is, however, worth noticing that the depth of the introduced attenuation by the proposed second-order filter is not only doubled as compared to its first-order counterpart, but, it has increased in its bandwidth. This is the direct consequence of doubling the reflection coefficient. Fig. 8*a* (trace c) represents the transmission coefficient of the proposed third-order absorptive filter. In-line with the predictions of the previous section, the depth of attenuation needs to be three times greater than its first-order counterpart, i.e. it needs to be in the region of 35.6 dB in the frequency range of operation of filter 2. However, the depth of the transmission coefficient of this filter stands at about 28.1 dB, which is about 7.5 dB below the value predicted by the theory. Given that the transmission coefficient of any reflective structure based on a 3-dB coupler is proportional to a weighted summation of its two reflective loads, the deviation of the attenuation depth of Fig. 8*a* (trace c) from the predicted 35.6 to 28.1 dB can be attributed to the difference of the reflection loads. This difference is likely due to manufacturing imperfections of the printed circuit boards.

The measured transmission coefficients of the cascade connection of the test filter, CER0367A and the proposed absorptive filters of Fig. 7 are presented in Fig. 8*b*. In particular, Fig. 8*b* (trace a) represents the measured transmission coefficient of the CTS filter (CER0367A) alone, while Figs. 8*b* (traces b–d) represent the measured transmission coefficients of the proposed first-, second- and third-order absorptive filters connected in cascade with the test filter, respectively. As evident, the introduction of first-, second- and third-order absorptive filters has resulted in the increase in the attenuation depth compared to the case of the test filter alone. In particular, the first-order absorptive filter has introduced increased attenuation into the stop-band of the main filter to the value of a minimum of 11.77 dB across the frequency range 2.17–2.27 GHz. In a similar fashion, the second-order absorptive filter has increased the level of the attenuation even further, and the depth of attenuation at the corresponding range of frequencies is now over 22.94 dB. The third-order absorptive filter offers an even further increase in the depth of attenuation – it now stands at a minimum of 27.38 dB. However, the introduction of the higher-order absorptive filters into the response of the test filter has not come without a penalty. The first-, second- and third-order absorptive filters have introduced an additional insertion loss into the pass-band of the test filter to the value of 0.72, 1.24 and 2.4 dB, respectively. These values compare favourably with the amount of additional insertion loss that is recorded for the case when higher-order absorptive filters are not realised using the presented theory, but by a simple cascade

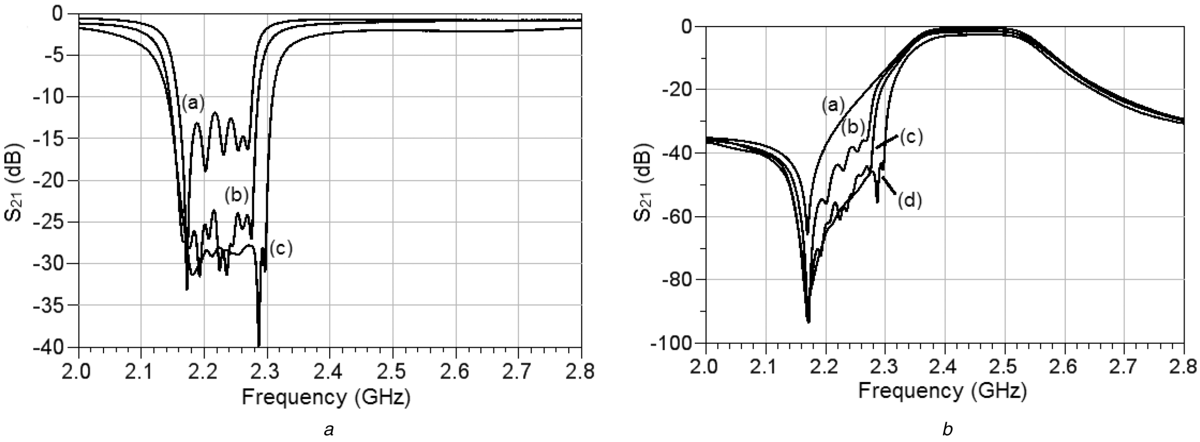


Fig. 8 *Transmission coefficients*

(a) Measured transmission coefficients of first-order absorptive filter, (trace a), proposed second-order absorptive filter (trace b) and proposed third-order absorptive filter, (trace c) and, ***(b)*** Measured transmission coefficients of CTS filter (CER0367A) alone (trace a), cascade connection of test filter with first-order absorptive filter (trace b), second-order absorptive filter (trace c) and third-order absorptive filter (trace d)

Table 2 Measured performance of complete filter of Fig. 7 (test filter connected in cascade with various absorptive filters)

	Minimum attenuation, dB	Maximum increase of insertion loss, dB
test filter in cascade with first-order absorptive filter	11.77	0.72
test filter in cascade with proposed second-order absorptive filter	22.94	1.24
test filter in cascade with second-order absorptive filter obtained by cascade connection of two first-order absorptive filters	23.54	1.64
test filter in cascade with proposed third-order absorptive filter	27.38	2.4
test filter in cascade with third-order absorptive filter obtained by cascade connection of three first-order absorptive filters	35.31	2.56

Table 3 Comparison of simulated and measured performances of complete filters of Fig. 7 (test filter connected in cascade with various absorptive filters)

	Minimum simulated attenuation, dB	Minimum measured attenuation, dB	Maximum increase of simulated insertion loss, dB	Maximum increase of measured insertion loss, dB
test filter in cascade with first-order absorptive filter	15.6	11.77	0.7	0.72
test filter in cascade with proposed second-order absorptive filter	29.65	22.94	1.16	1.24
test filter in cascade with proposed third-order absorptive filter	46	27.38	2.16	2.4

Table 4 Performance comparison of different absorptive filter technologies

	Rauscher [4]	Li *et al.* [6]	Jachowski [9]	This work (third-order absorptive filter)
operating frequency, GHz	10	1.525	2	2.28
Bandwidth, %	0.7	0.65	0.5	4.38
min. attenuation, dB	40	45	45	46

connection of absorptive filters of the first order in series with the test filter. Table 2 summarises these findings.

For completeness, the highlights of the simulated and measured results of the proposed absorptive filters are shown in Table 3, whereas a simulated performance comparison of the proposed third-order absorptive filter to the previously published studies is given in Table 4.

4 Conclusions

In this paper, a design methodology and equations necessary for the introduction of low-loss high-order reflective-type, absorptive filters are introduced and described in detail. The presented derived equations allow for a straightforward design of high-order absorptive filters. As an experimental verification, several absorptive filters have been designed, tested and their effect on the performance of a test filter evaluated. It was shown that the proposed filters greatly increase the depth of attenuation of the transmission coefficient and, in the case of a third-order absorptive circuit, the depth of attenuation of over 27 dB, is achieved.

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6. Appendix

In this section, full derivation of the conditions needed for the third-order absorptive filter is presented. Substituting $n = 3$ in (18), the input admittance in the form of a polynomial of a third order is obtained, from which the transmission coefficient (10) can be inferred. Solving (10) for multiple and real roots yields a triple zero, given by (12), with a condition that the discriminant (11) is zero. Solving (11) one obtains a quart-quadratic equation in k_{11}^4, given by

$$Ak_{11}^8 + Bk_{11}^4 + C = 0 \tag{27}$$

where $A = -4k_{22}^4$, $B = -8Z_0^2k_{12}^4k_{22}^2 + 20Z_0^2k_{12}^2k_{22}^2k_{21}^2 + Z_0^2k_{22}^2k_{21}^2$ and $C = -4Z_0^4k_{12}^8 - 4Z_0^4k_{12}^2k_{21}^4 - 12Z_0^4k_{12}^6k_{21}^2 - 12Z_0^4k_{12}^4k_{21}^4$. The double zero in k_{11}^4 is achieved at

$$k_{11}^4 = -\frac{B}{2A} = \frac{-8Z_0^2k_{12}^4k_{22}^2 + 20Z_0^2k_{12}^2k_{22}^2k_{21}^2 + Z_0^2k_{22}^2k_{21}^4}{8k_{22}^4} \tag{28}$$

with a condition that the discriminant of (27), $\Delta_1 = B^2 - 4AC$, disappears. This condition yields a third-order polynomial in k_{12}^2, given by

$$Dk_{12}^6 + Ek_{12}^4 + Fk_{12}^2 + G = 0 \tag{29}$$

where $D = -512$, $E = 192k_{21}^2$, $F = -24k_{21}^2$ and $G = k_{21}^6$. The triple zero of (29) is achieved at

$$k_{12}^2 = -\frac{E}{3D} = \frac{1}{8}k_{21}^2 \tag{30}$$

provided that the discriminant of (29) disappears. It can be shown that the discriminant of (29) is always equal to zero, regardless of the value assigned to k_{21}^2. This infers that k_{21}^2 can be used as a parameter. Substituting (30) into (28), one finds the expression for k_{11}^2 where k_{22} and k_{21}^2 are used as parameters

$$k_{11}^2 = \sqrt{\frac{27}{64}\frac{Z_0k_{21}^2}{k_{22}}} \tag{31}$$

The relationship between k_{22} and other impedance transformers is found from (12). Imposing that the triple zero of (10) occurs at Z_0, (12) gives the following relationship for k_{22}

$$k_{22} = \frac{\sqrt{27}}{3}Z_0 \tag{32}$$

Substituting (32) into (31), the expression for k_{11} now becomes

$$k_{11}^2 = \frac{3}{8}k_{21}^2 \tag{33}$$

It can be seen that expressions (29), (31) and (32) are identical to those provided by (13).